Round-bounded control of parameterized systems

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Midi de Move

25/10/2018

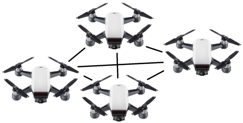
Parameterized system?

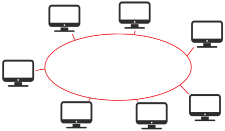
Paremeterized System

Distributed system with no fixed number of processes

Examples

Mobile networks, distributed algorithms, drone swarms, ...





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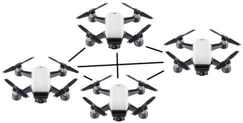

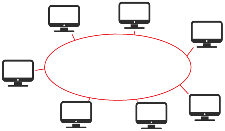

⇒ Should work for any number of processes!

Reachability vs Control

Systems interact with environment: User inputs, Drone sensors, ...

Reachability vs Control

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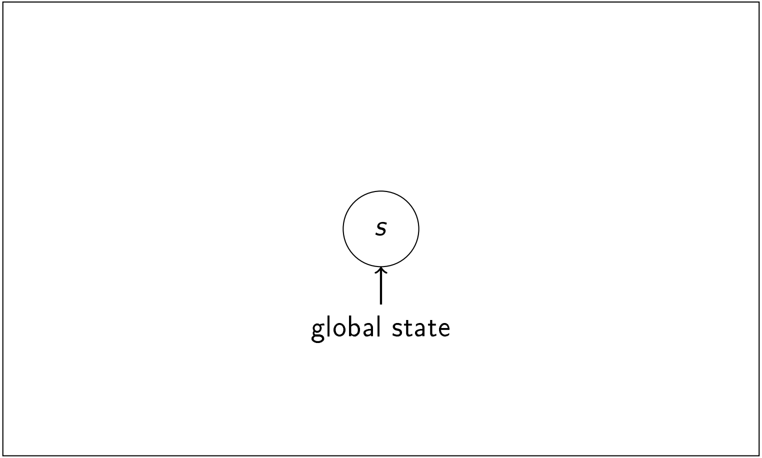
Reachability

Is there a correct behavior?

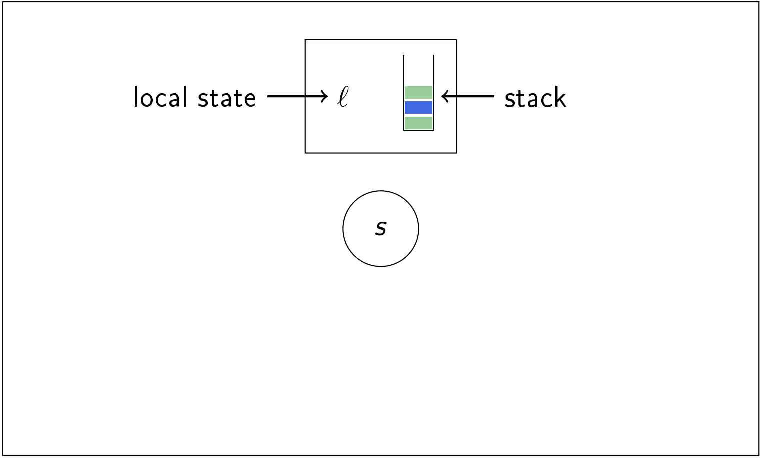
Control

Can we force every behavior to be correct?

Parameterized Pushdown Systems



Parameterized Pushdown Systems



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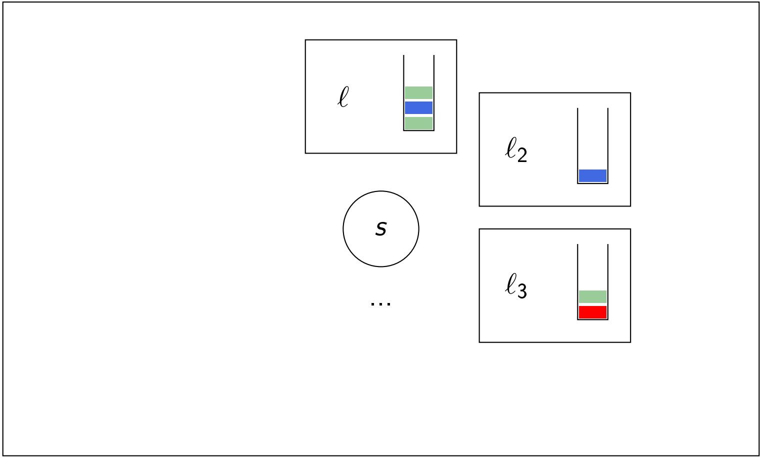

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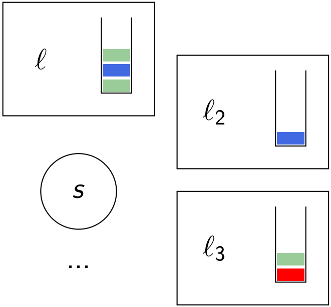
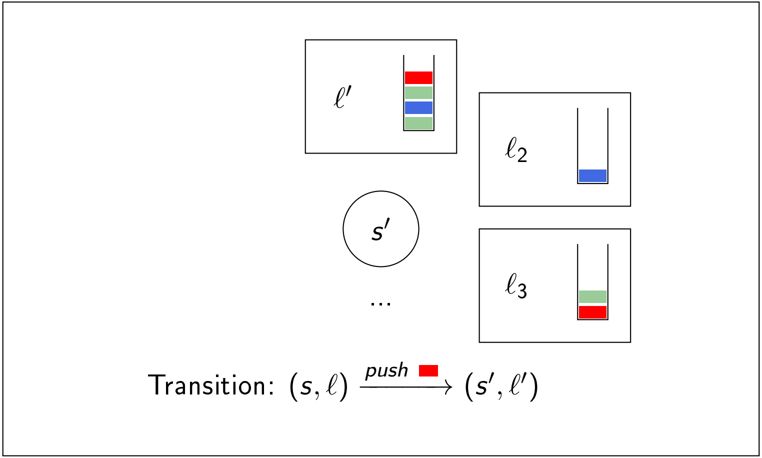


Transition: $(s, \ell) \xrightarrow{push\ 🟥} (s', \ell')$

Parameterized Pushdown Systems



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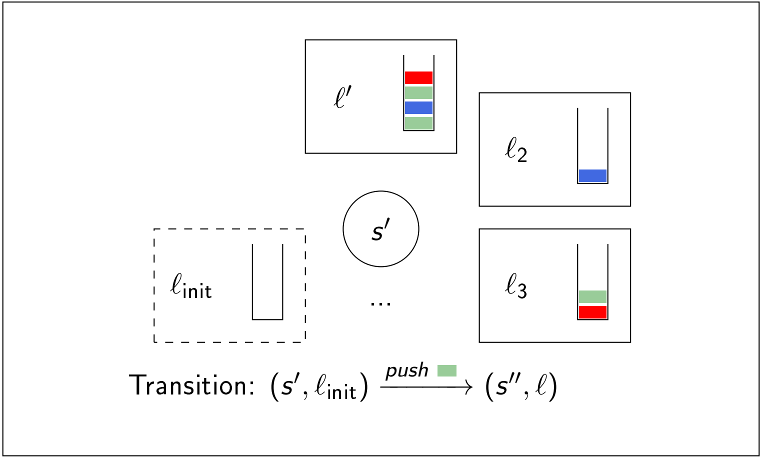
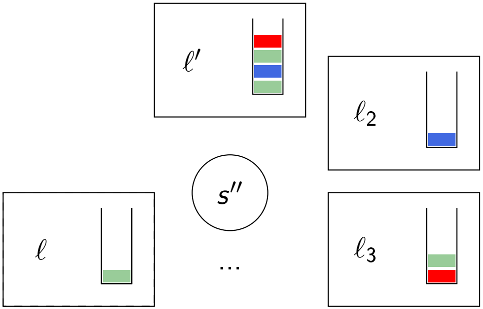


$$\text{Transition: } (s', \ell_{\text{init}}) \xrightarrow{\textit{push}\ \text{🟩}} (s'', \ell)$$

Parameterized Pushdown Systems



Transition: $(s', \ell_{\mathrm{init}}) \xrightarrow{\mathit{push}\ 🟩} (s'', \ell)$

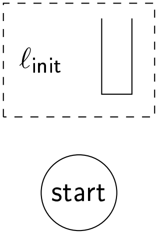
Example: Scheduler model (1/2)

Behavior we want to model:

1. Scheduler sends requests and starts processes,
2. Then each process performs the requested tasks,
3. Then each process stops.

Example: Scheduler model (2/2)

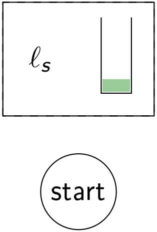
1. Scheduler sends requests and starts processes



$$(\text{start}, \ell_{\text{init}}) \xrightarrow{\mathit{push}\ \text{🟩}} (\text{start}, \ell_s)$$

Example: Scheduler model (2/2)

1. Scheduler sends requests and starts processes



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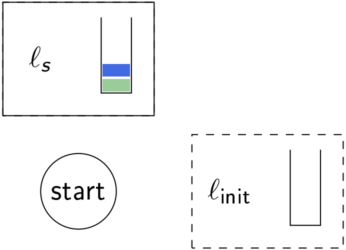

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…

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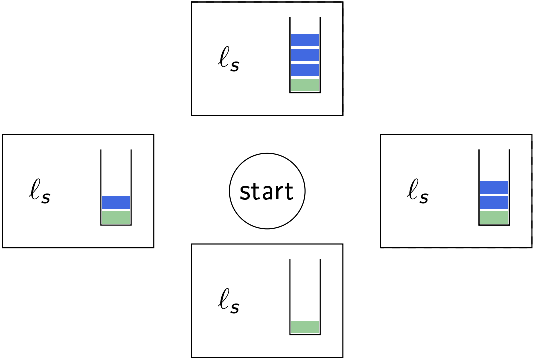
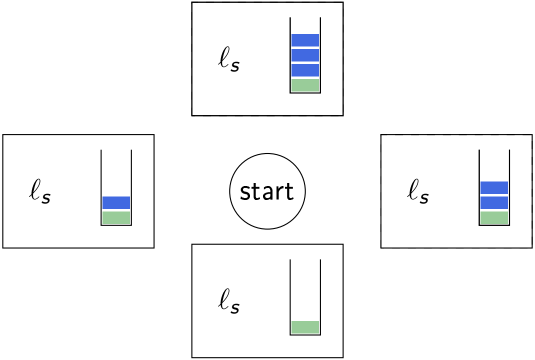


Example: Scheduler model (2/2)

2. Each process performs the requested tasks



$$(\mathsf{start}, \ell_s) \xrightarrow{-} (\mathsf{task}, \ell_t)$$

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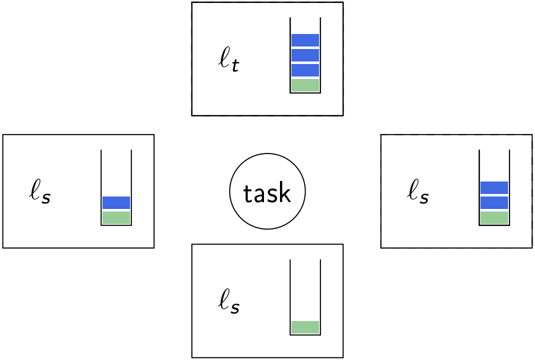

$$(\text{task}, \ell_t) \xrightarrow{pop\ \blacksquare} (\text{task}, \ell_t)$$

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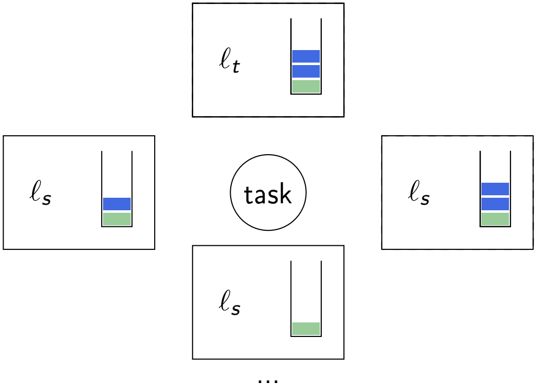
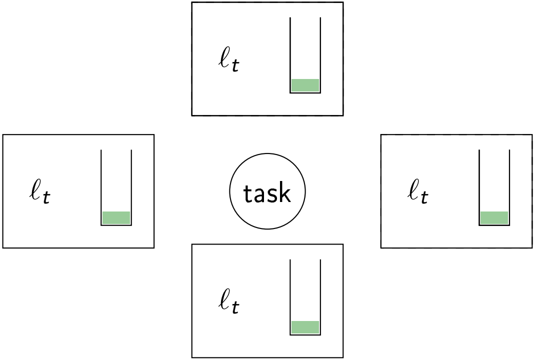


Example: Scheduler model (2/2)

3. Each process stops



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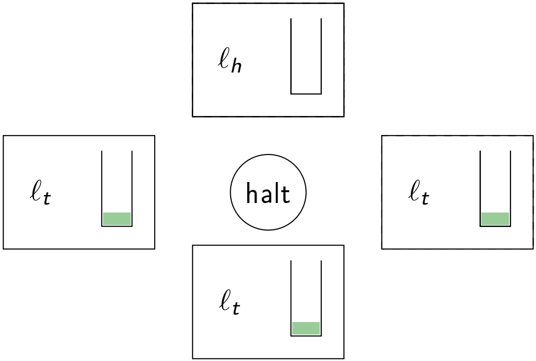

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Round-bounded behaviors

Even Reachability is undecidable!

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Restriction: rounds [La Torre et al., 2010]

One round:

✓ 1 1 1 1 2 2 2 3 3 3 3 3 4 5 5 5 ...

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Round-bounded behaviors

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One round:

✓ 1 1 1 1 2 2 2 3 3 3 3 3 4 5 5 5 ...

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✗ 1 1 1 2 2 3 3 1 4 4 6 ...

Round-bounded behaviors

Even Reachability is undecidable!

Restriction: rounds [La Torre et al., 2010]

One round:

- ✓ 1 1 1 1 2 2 2 3 3 3 3 3 4 5 5 5 ...
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- X 1 1 1 2 2 3 3 1 4 4 6 ...

N rounds:

1 1 1 2 3 4 4 | 1 3 3 4 5 | 3 3 5 6 6 6 7 | ...

Round-bounded behaviors

Even Reachability is undecidable!

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N rounds:

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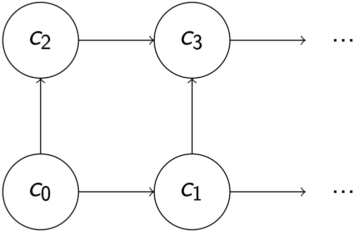
Round-bounded Reachability is PSPACE-complete.

Control: Parameterized Pushdown Games



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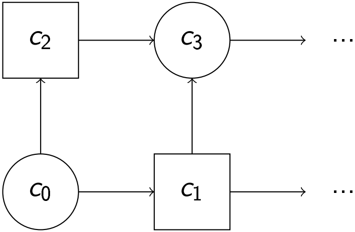

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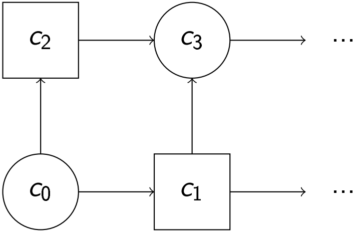

Control problem

Is there a winning strategy for the Controller?

Main result

Decidable, but inherently non-elementary.

Decidability

Reduction to *phase-bounded multi-pushdown games*
[Atig et al., 2017]

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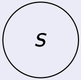
Multi-pushdown systems



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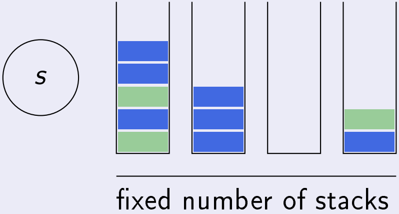

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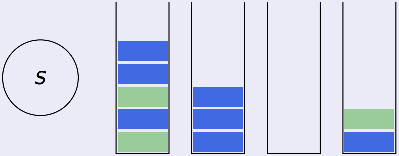

$$s \xrightarrow{push\ 2\ 🟥} s'$$

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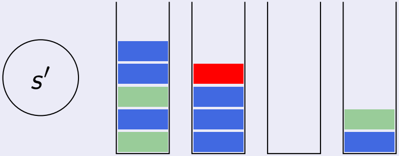

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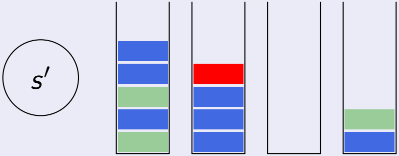
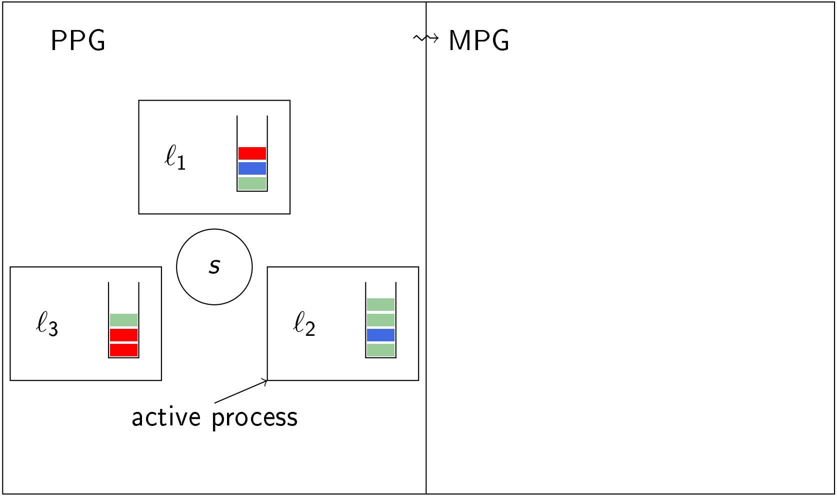


$$s \xrightarrow{\textit{push}\ 2\ 🟥} s'$$

Phase-bounded

Phase: pop only from one stack, push unrestricted.

Sketch of the reduction - encoding configurations



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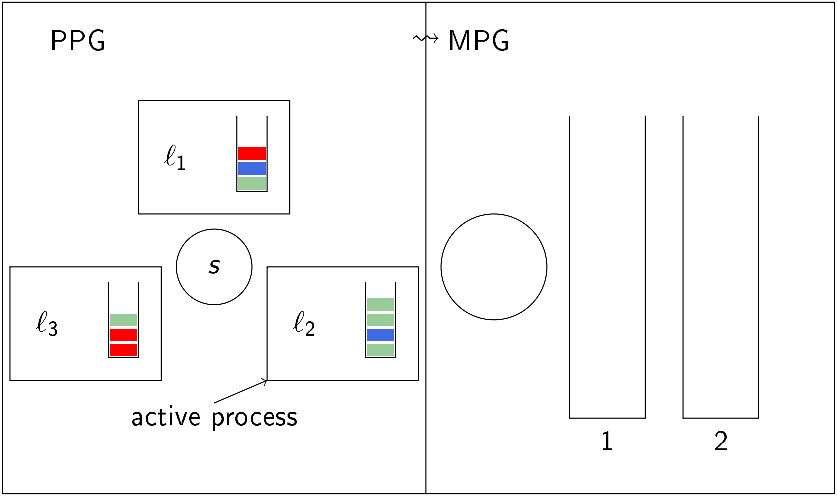

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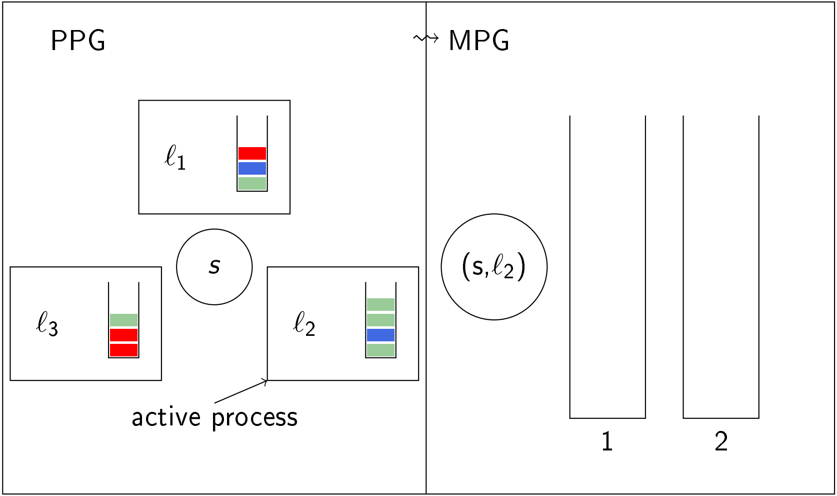

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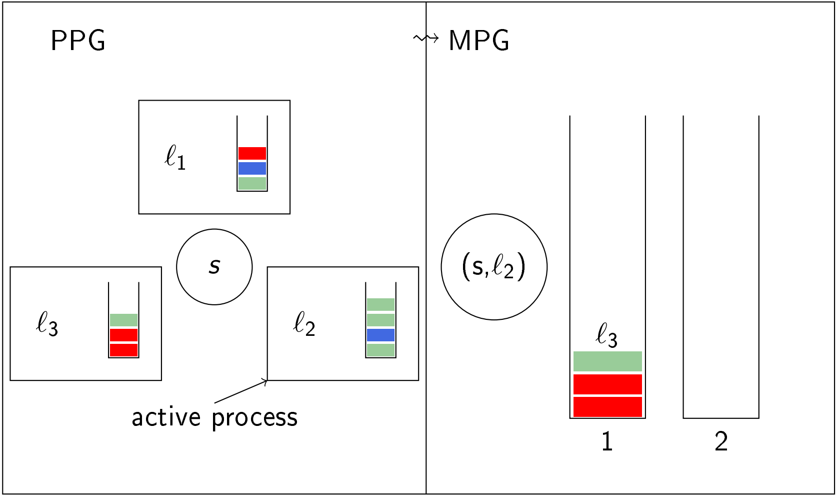

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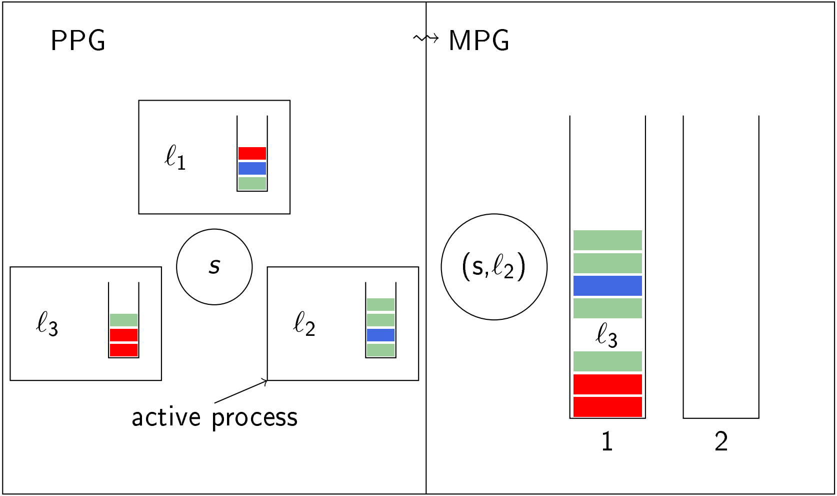

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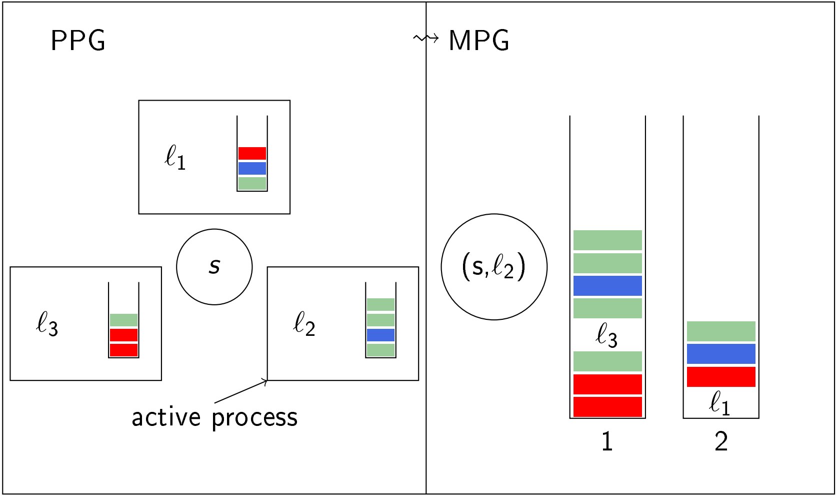
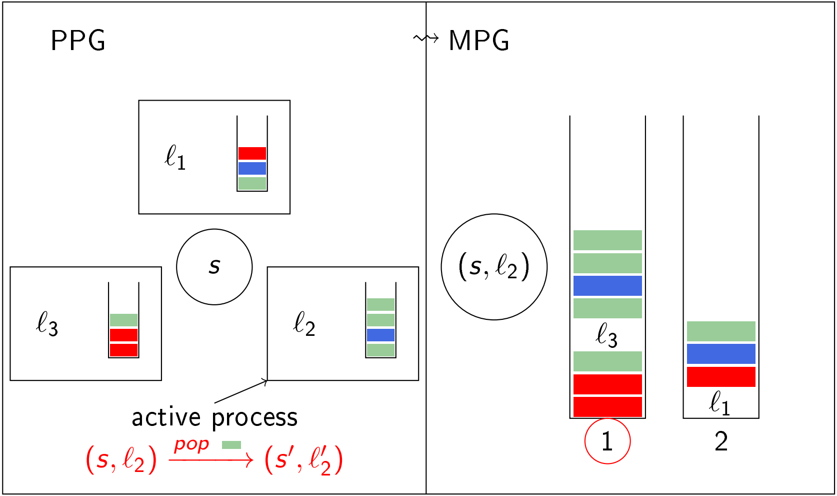


Sketch of the reduction - simulating transitions



$(s, \ell_2) \xrightarrow{pop\ 🟩} (s', \ell'_2)$

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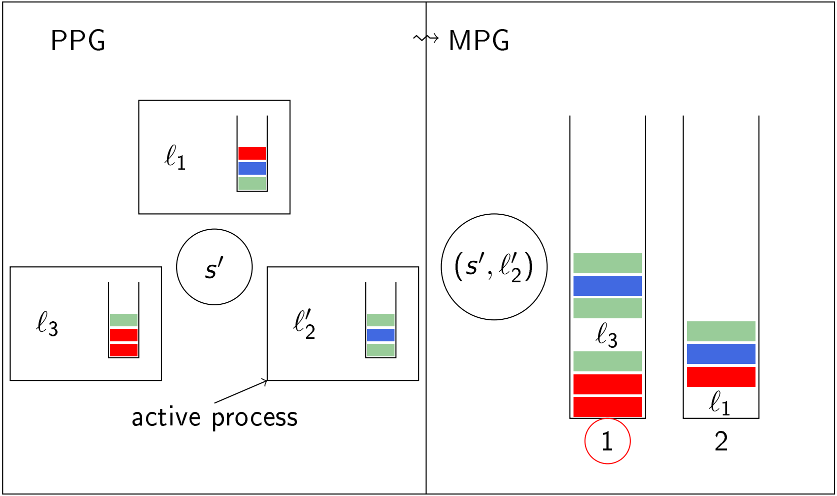

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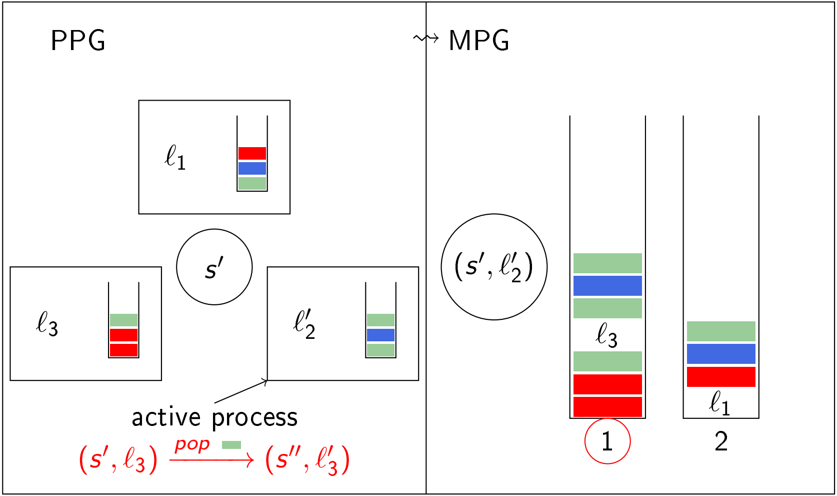

active process

$(s', \ell_3) \xrightarrow{\text{pop 🟩}} (s'', \ell'_3)$

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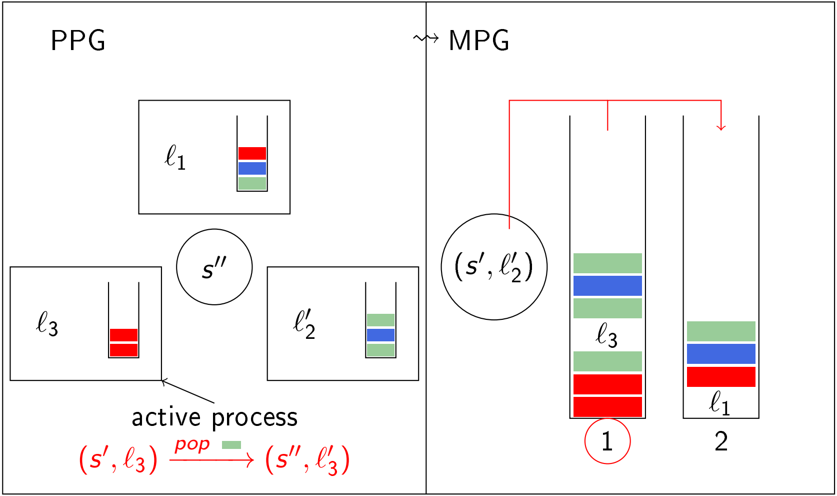

$$(s', \ell_3) \xrightarrow{pop\ 🟩} (s'', \ell_3')$$

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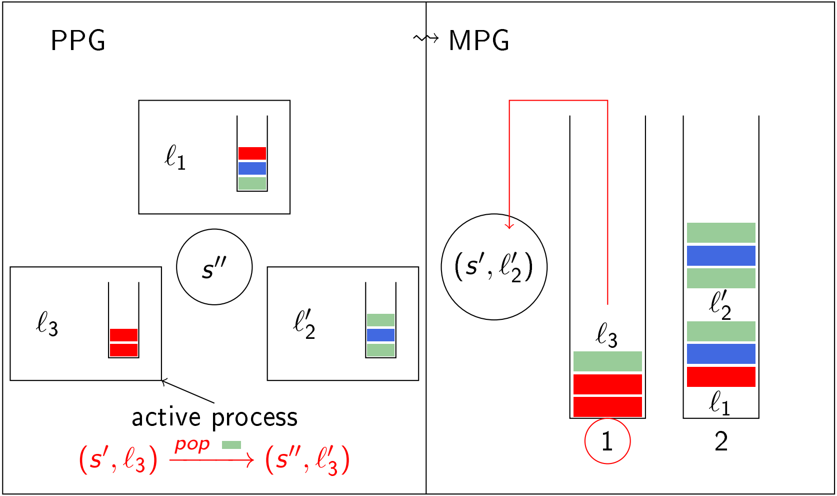

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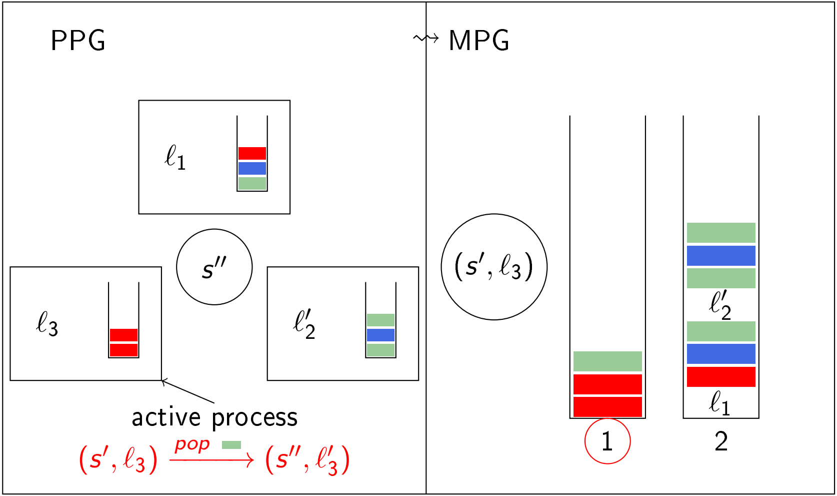

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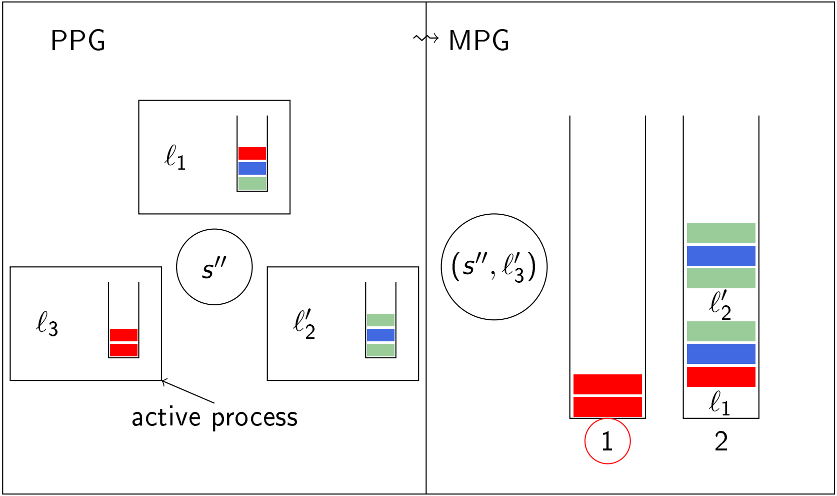

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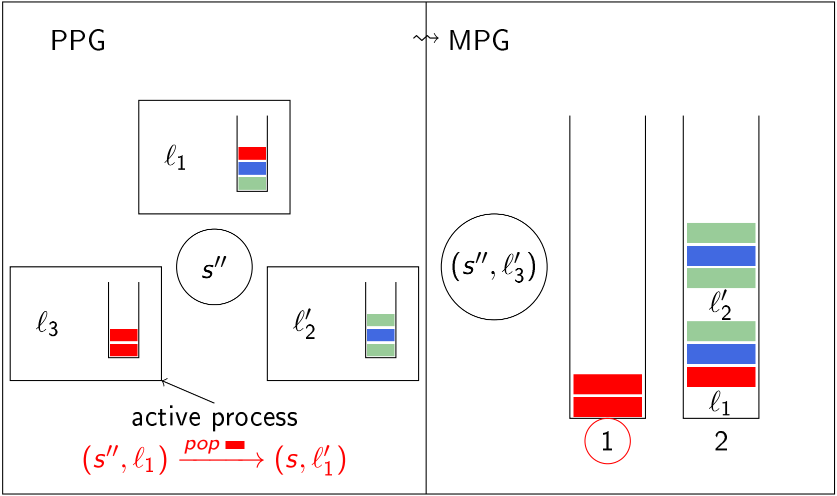

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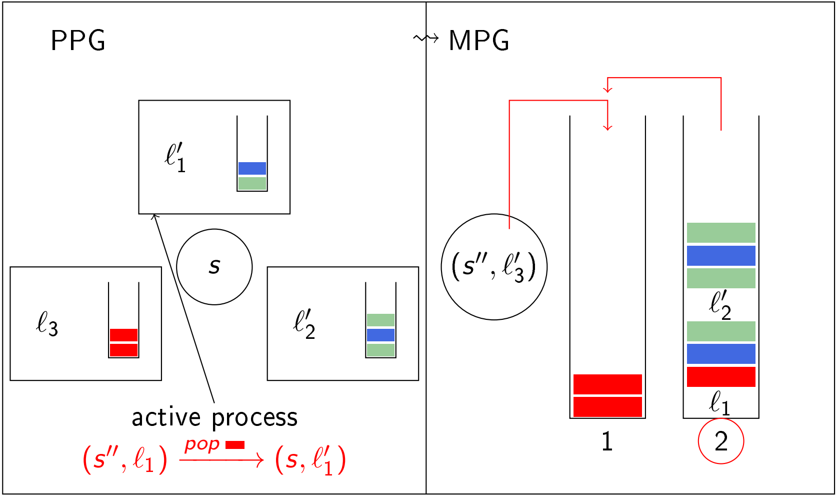

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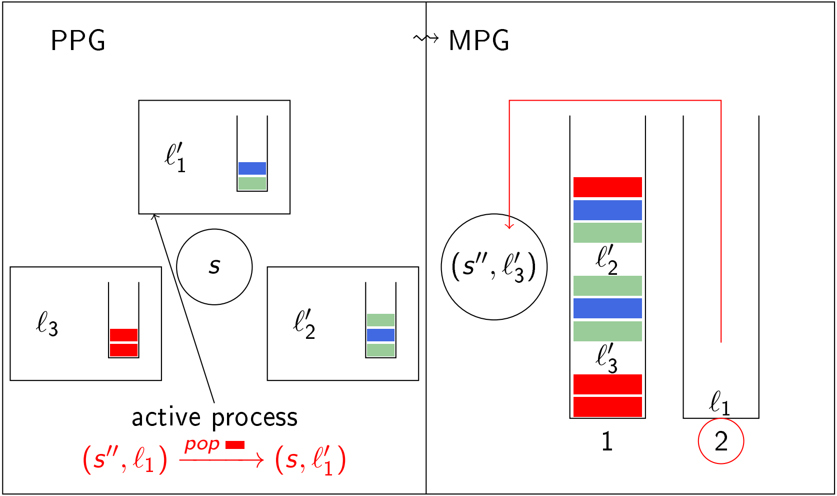

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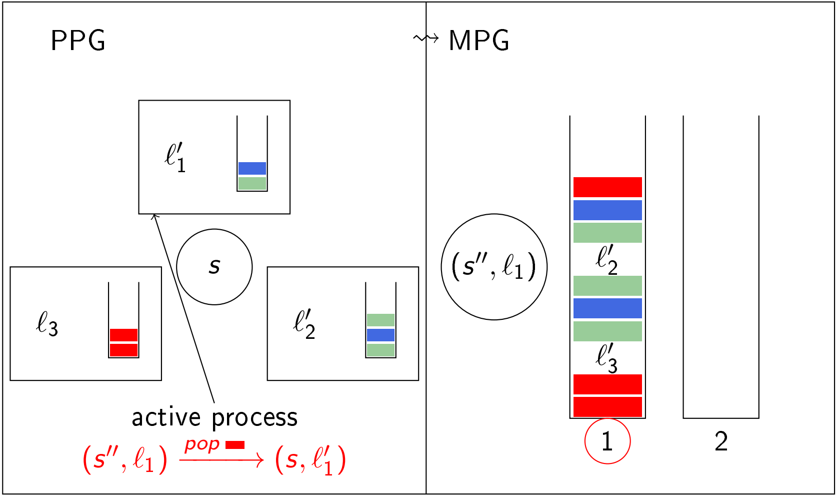

active process

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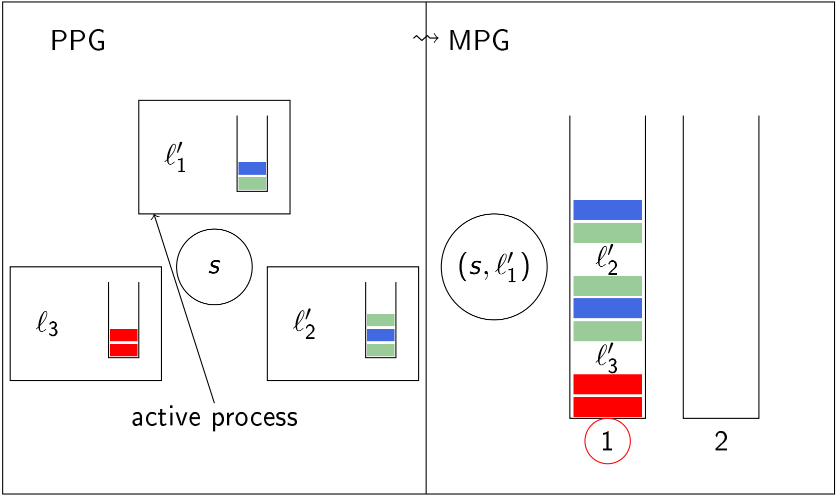

Hardness (1/2)

Reduction of satisfiability of FO($<$) on finite words. [Stockmeyer, 1970]

Example: $\exists x.\forall y.\mathrm{a}(x) \wedge (\mathrm{b}(y) \Rightarrow y < x)$

✓ a b c b a c c a

✗ a a b a b c

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✓ a b c b a c c a

X a a b a b c

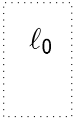
Syntax

$$\begin{aligned} t &::= \mathrm{a}(x) \mid x < y \mid x = y \\ \varphi &::= t \mid \neg t \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x.\varphi \mid \forall x.\varphi \end{aligned}$$

with $x, y \in \mathrm{Var}$ and $a \in \Sigma$.

Hardness (2/2)

Step 1: Controller creates the witness



$$\exists x. \forall y. \mathrm{a}(x) \land (\neg \mathrm{b}(y) \lor y < x)$$

Hardness (2/2)

Step 1: Controller creates the witness

a	b	c	b	a	c	c	a

$$\exists x . \forall y . \mathrm{a}(x) \wedge (\neg \mathrm{b}(y) \vee y < x)$$

Hardness (2/2)

Step 2: Controller and Environment assign values to variables

a	b	c	b	a	c	c	a

$$\underline{\exists x}.\forall y.\mathrm{a}(x) \wedge (\neg \mathrm{b}(y) \vee y < x)$$

Hardness (2/2)

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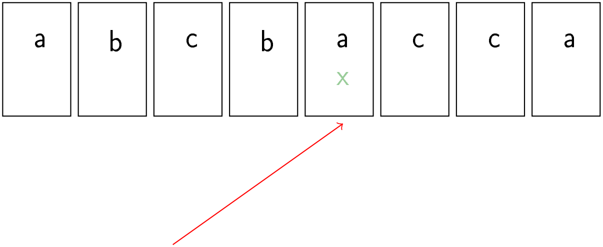

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				x			

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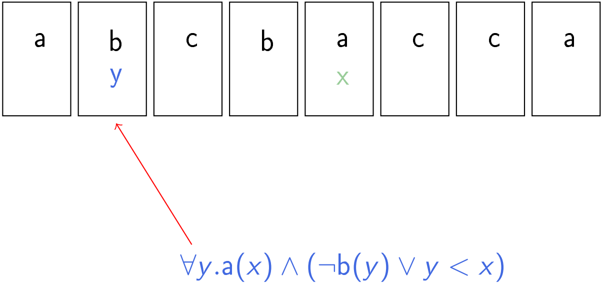


$$\forall y.\mathrm{a}(x) \wedge (\neg \mathrm{b}(y) \vee y < x)$$

Hardness (2/2)

Step 3: Disjunctions and Conjunctions

a	b y	c	b	a x	c	c	a

$$\mathsf{a}(x) \underline{\wedge} (\neg \mathsf{b}(y) \vee y < x)$$

Hardness (2/2)

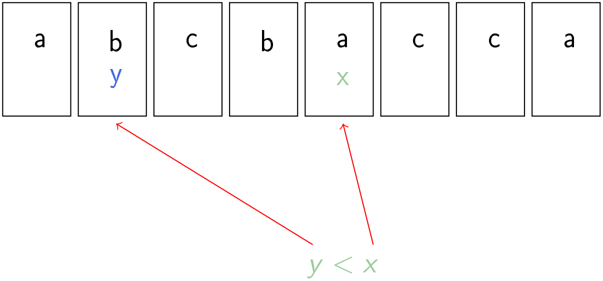
Step 3: Disjunctions and Conjunctions

a	b	c	b	a	c	c	a
	y			x			

$$\neg \mathrm{b}(y) \underline{\lor} y < x$$

Hardness (2/2)

Step 4: Checking the remaining term



Hardness (2/2)

Step 4: Checking the remaining term

a	b	c	b	a	c	c	a
	y			x			

Win!

Conclusion

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Thanks for listening, questions?