Synthesis for Parameterized Systems

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Master class

12 September 2019

The context

Distributed systems everywhere:

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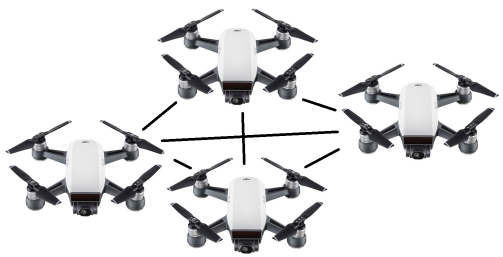

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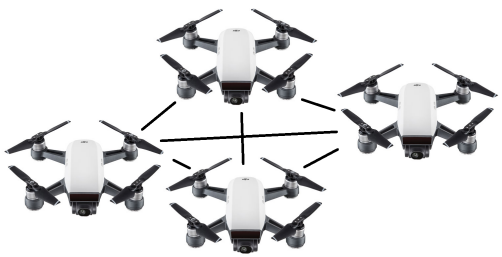

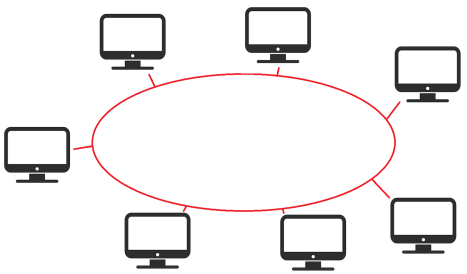

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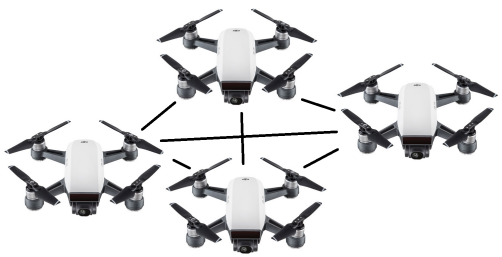

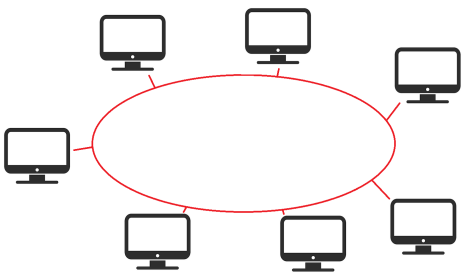

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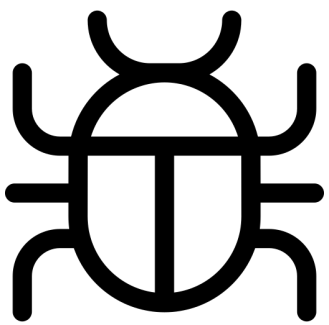

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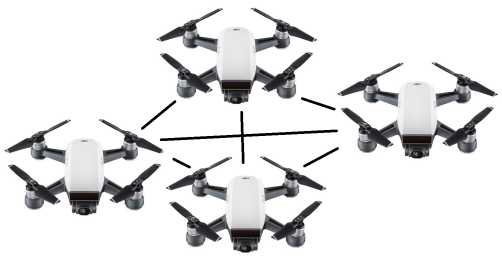

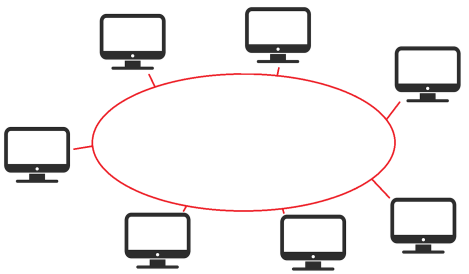

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Our focus

Parameterized Systems

A *parameterized system* is a distributed system with no fixed number of processes.

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Processes interact with an uncontrollable environment.

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Specification

The system as a whole must complete some given task.

Synthesis

Synthesis problem

Given a specification φ, can we build a system S such that $(S||Env) \models \varphi$?

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Motivations

- Bug-free by design
- Saving time

Problems

- Uncontrollable environment
- Unbounded number of processes

Behaviors of the system

Datawords

Words over $\Sigma \times \mathcal{D}$ with Σ a finite alphabet and $\mathcal{D}$ an infinite set.

Example over $\{req, ack\} \times \mathbb{N}$

$w = (req, 1)(req, 3)(ack, 1)(req, 6)(ack, 6)(ack, 3)$

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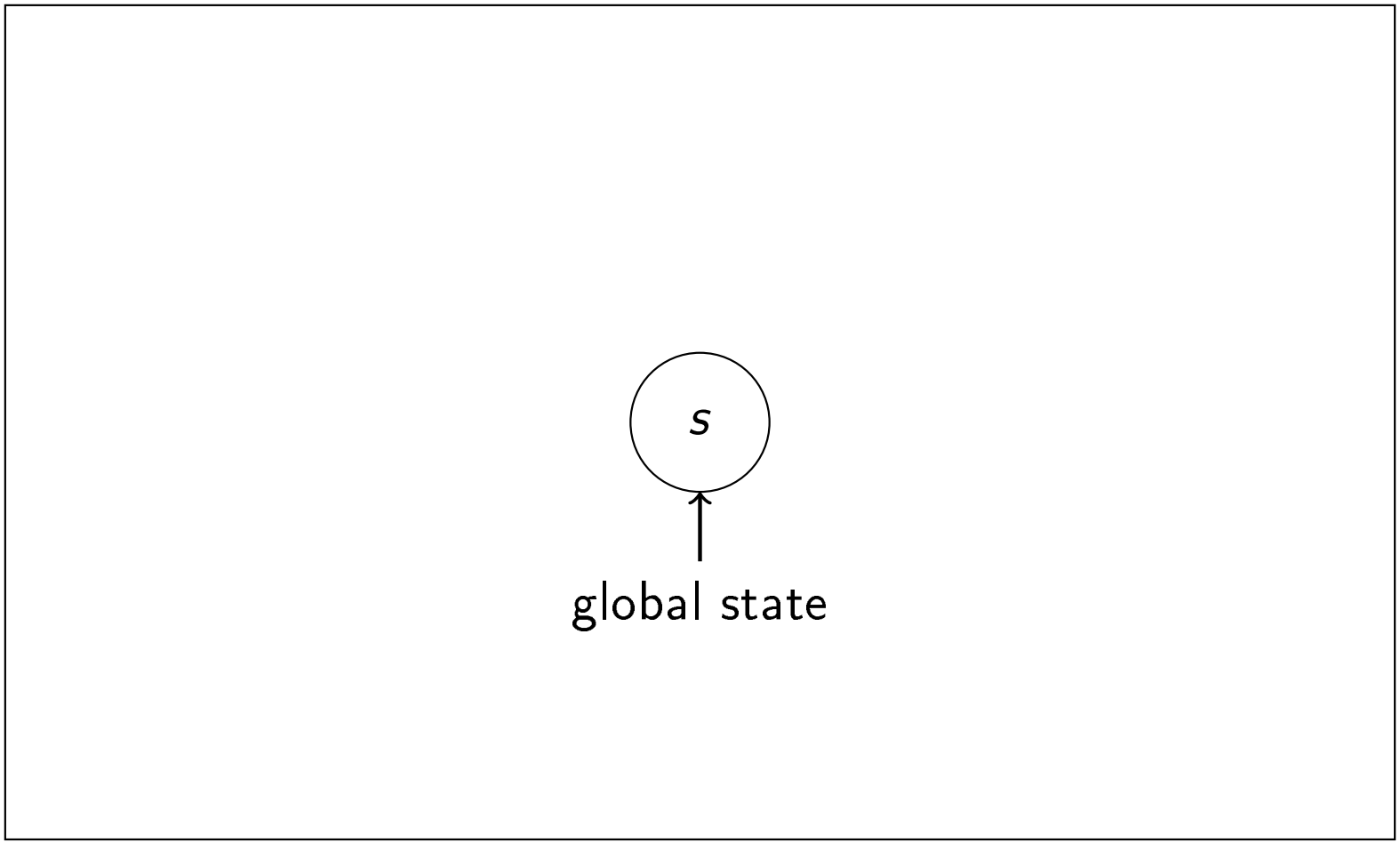
Specification

A specification is a set of *correct* datawords.

$\Rightarrow$ Specification as an automaton or a logic formula.

Parameterized Pushdown Systems



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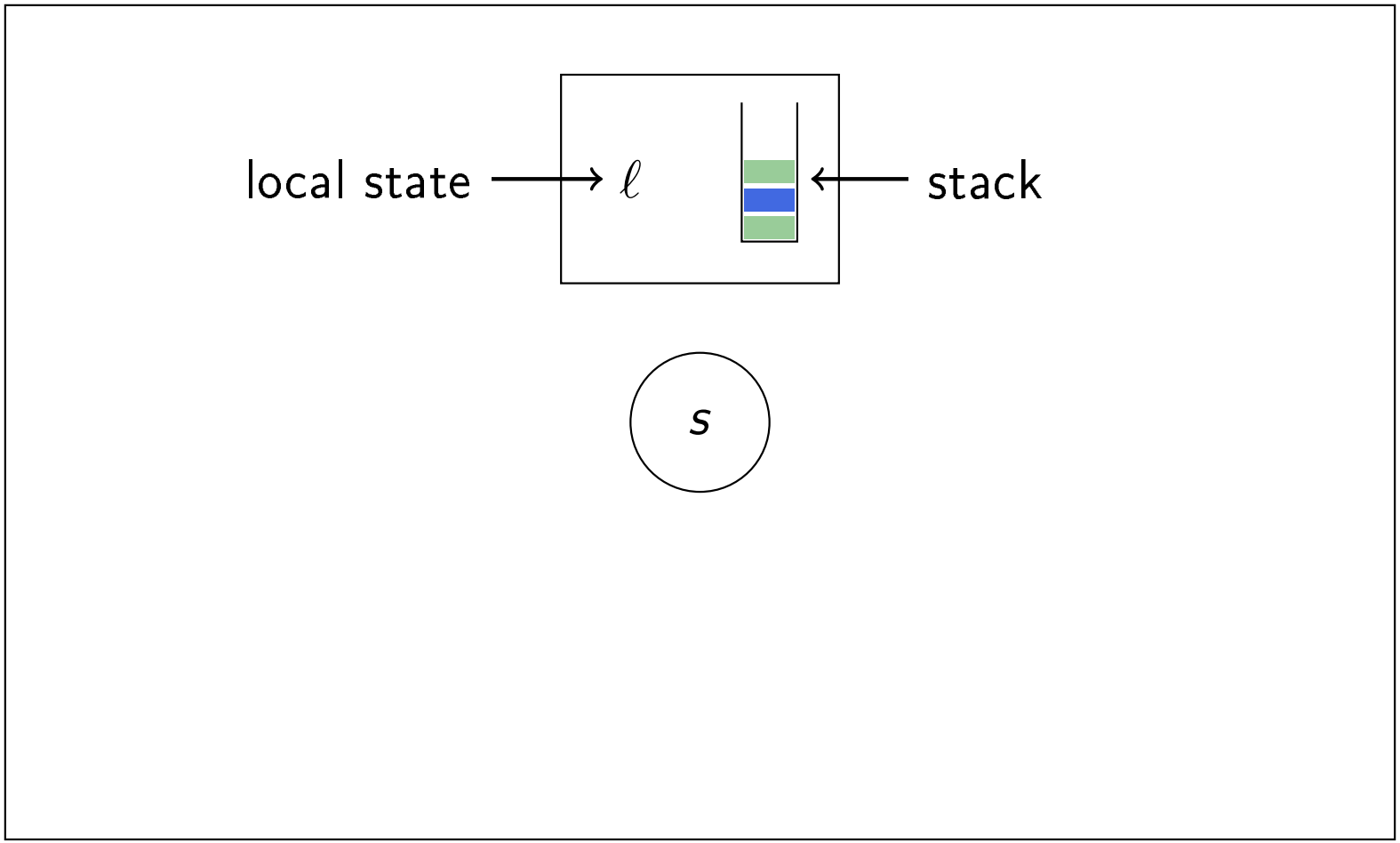

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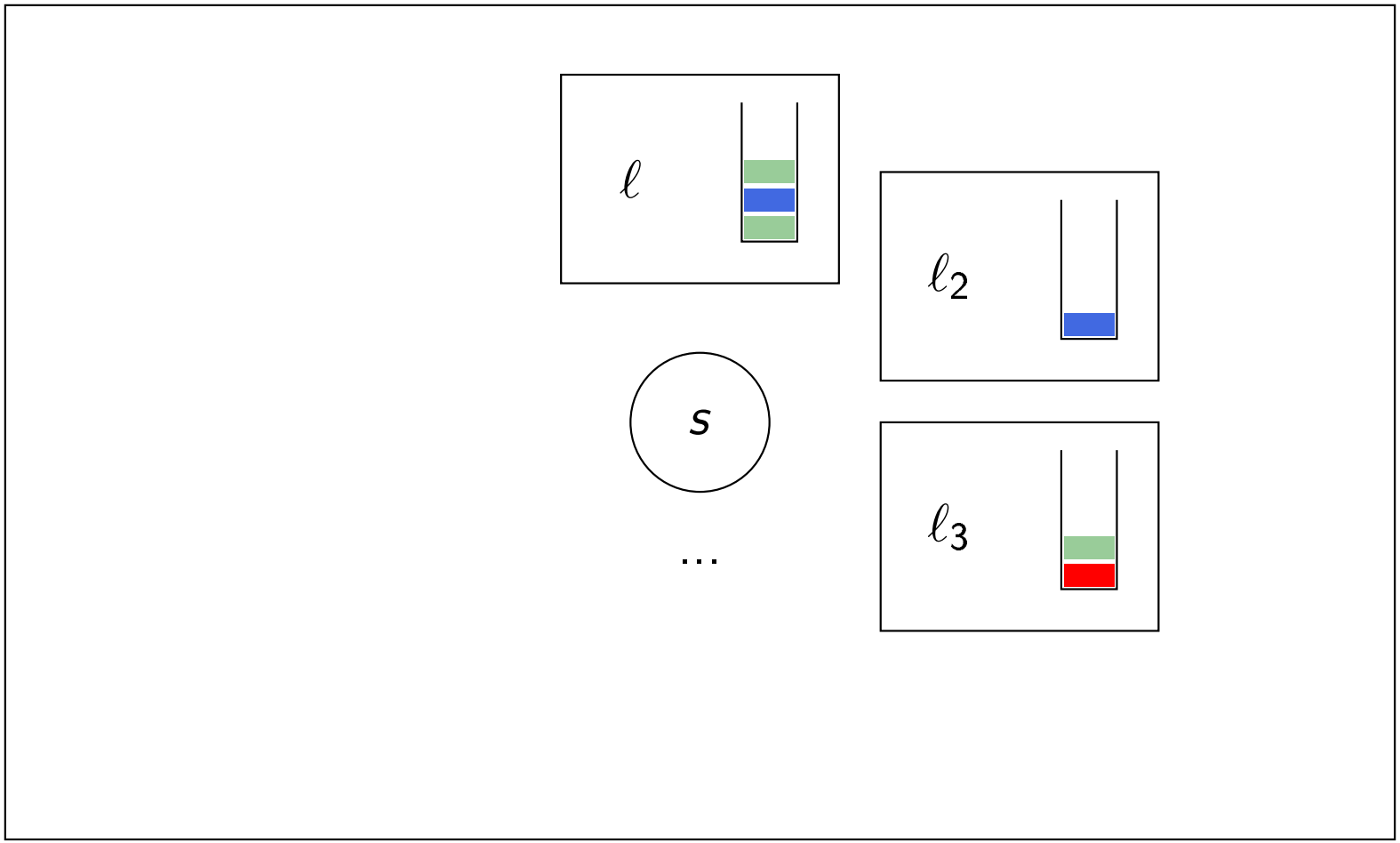

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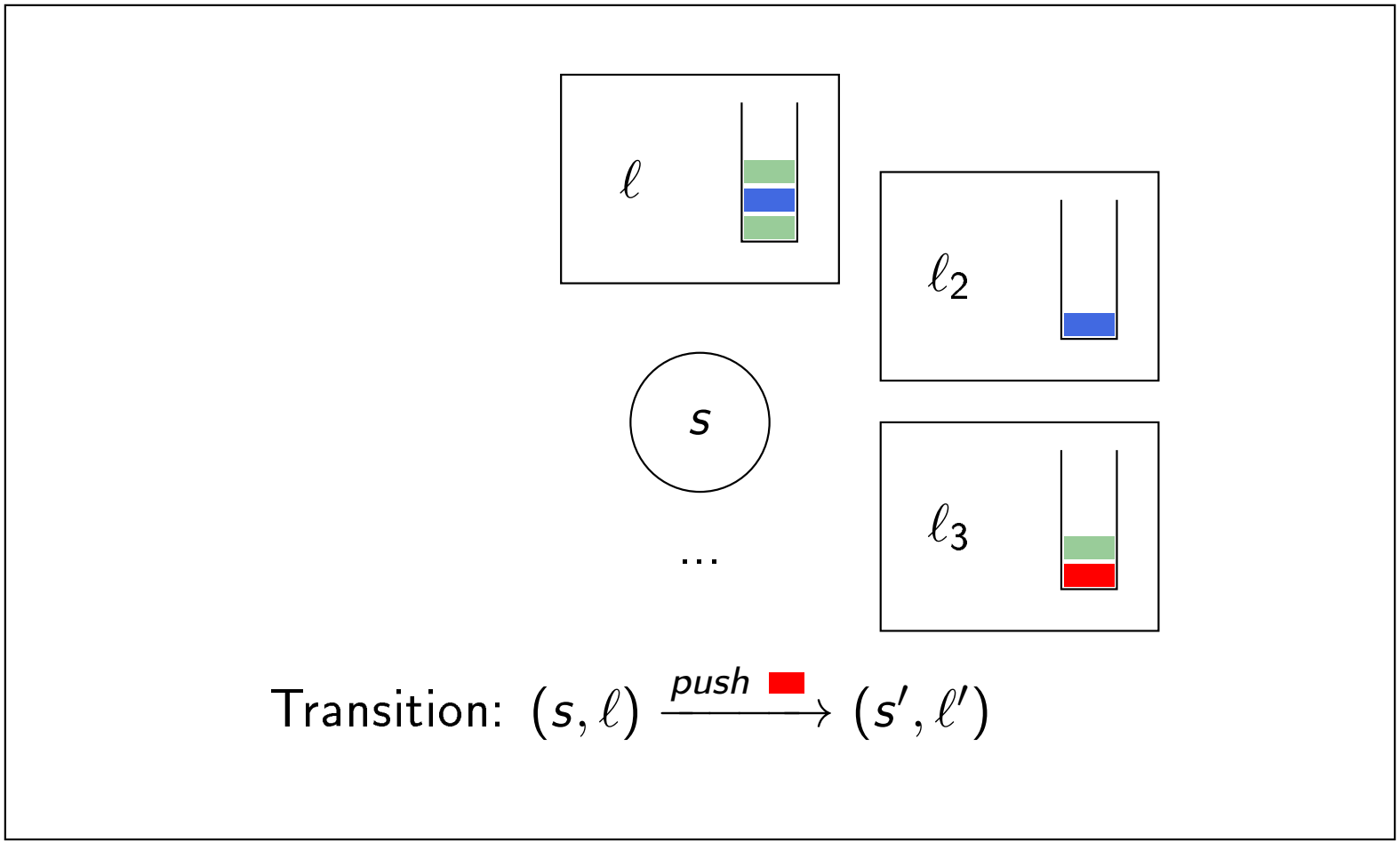

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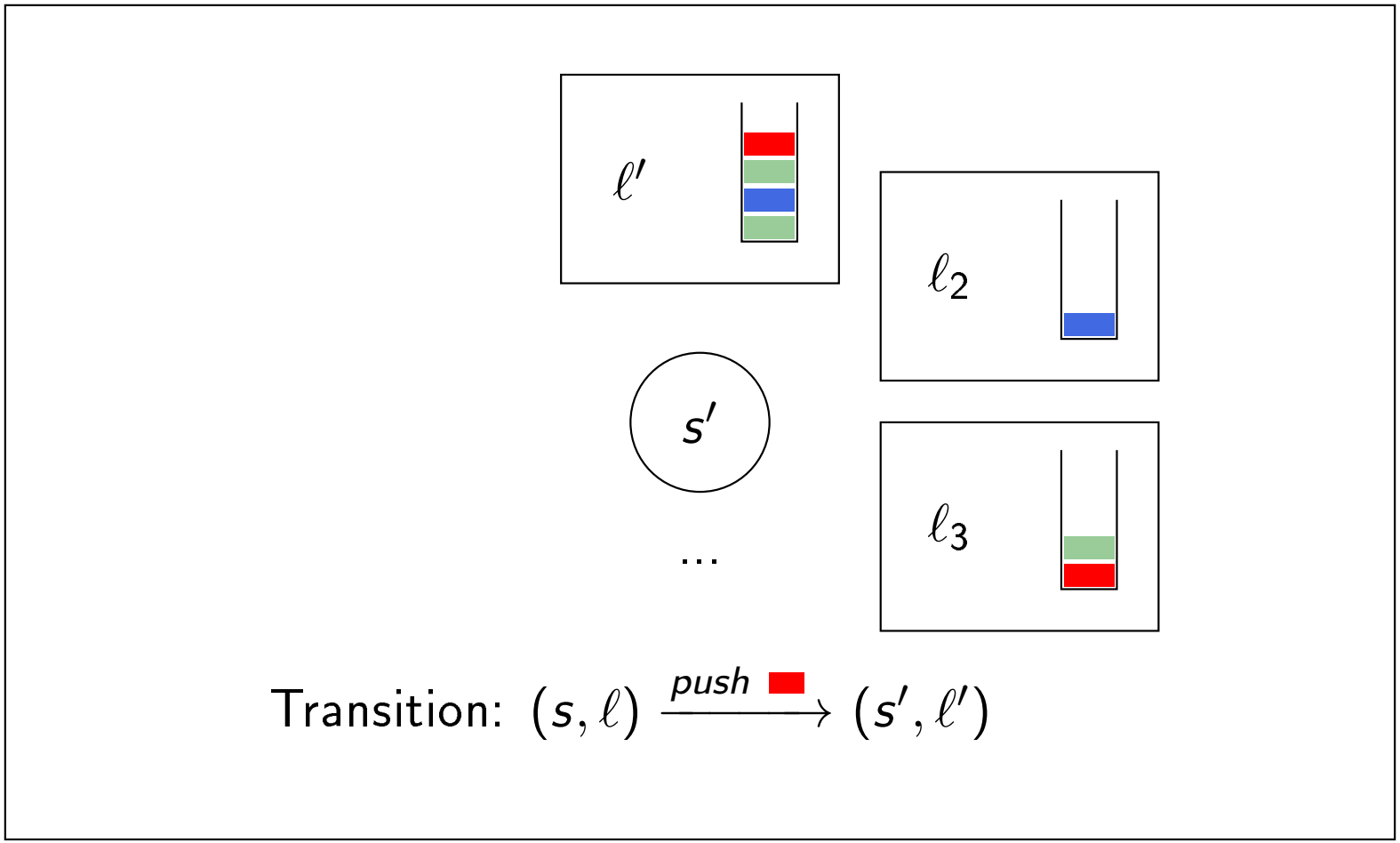

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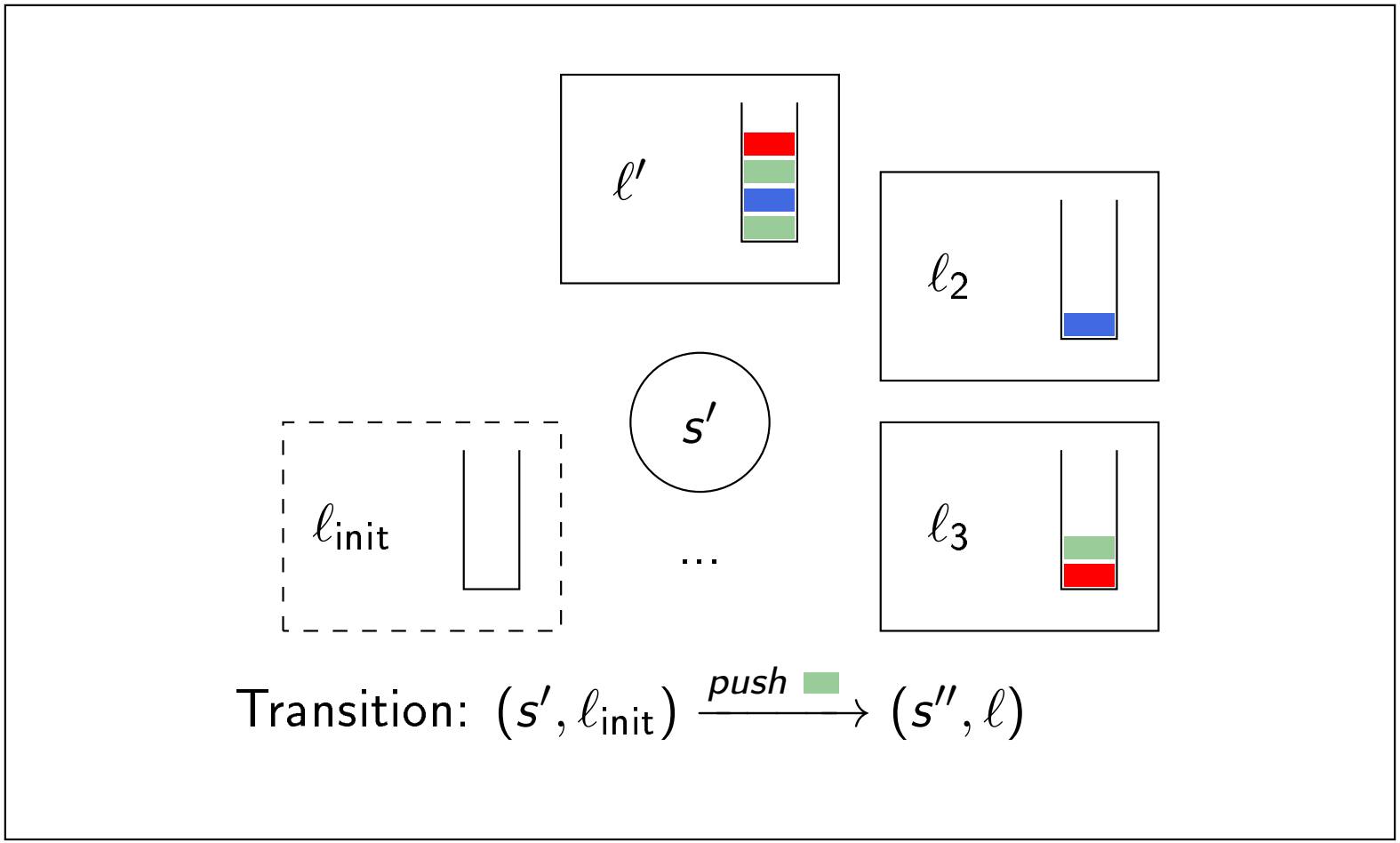

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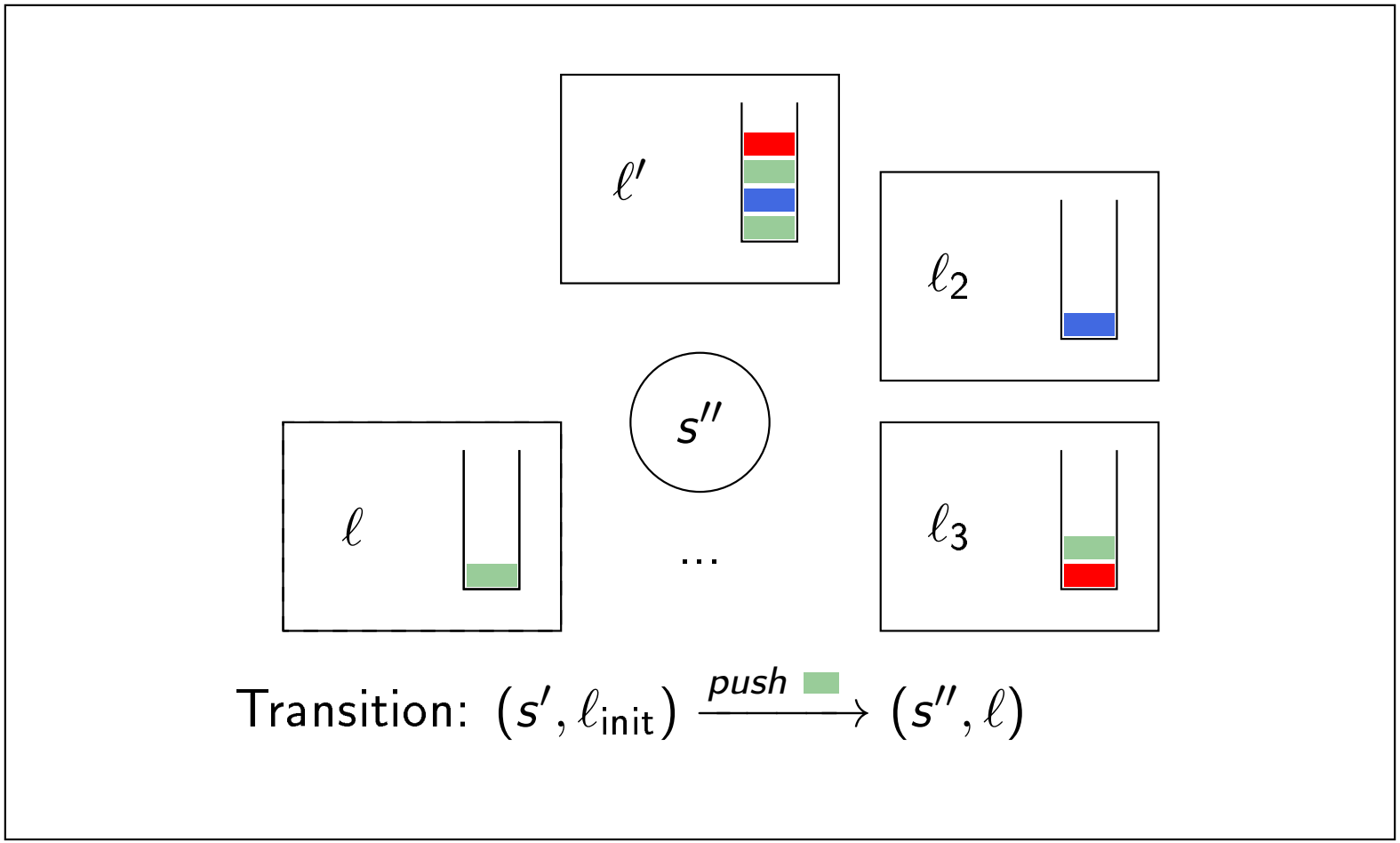

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Restriction: round-bounded behaviors [La Torre et al., 2010]

1 round:

- ✓ 1 1 1 1 2 2 2 3 3 3 3 3 4 5 5 5 ...
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Our results

- Emptiness problem is PSPACE-complete
- Synthesis problem is decidable (inherently non-elementary)

First Order Logic

Let Σ be a finite alphabet.

FO

First order formula:

$$\varphi ::= x = y \mid a(x) \mid \neg\varphi \mid \varphi \vee \varphi' \mid \varphi \wedge \varphi' \mid \forall x.\varphi \mid \exists x.\varphi \mid$$
$$x \sim y \mid x < y \mid x = y + 1$$

with $x, y, \ldots$ variables and $a, \ldots \in \Sigma$

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Example

$\varphi = \forall x.(req(x) \Rightarrow \exists y.(y \sim x \wedge y > x \wedge ack(y)))$

Synthesis problem and results

$$\Sigma = \Sigma_{sys} \uplus \Sigma_{env}$$

Synthesis problem viewed as a game

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Thank you for your attention!