Synthesis of Distributed Implementations from Centralized Specifications

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Joint work in progress with Daniel Hausmann and Nir Piterman

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What is synthesis actually?

Synthesis Problem

Input: A specification φ
Output: A program P satisfying φ

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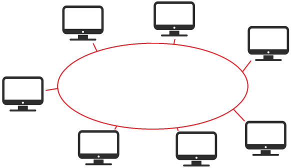
- Great if possible, but very hard.
- Distributed systems makes it even harder!
- Specifications are centralized, programs are distributed.



Example of a distributed protocol

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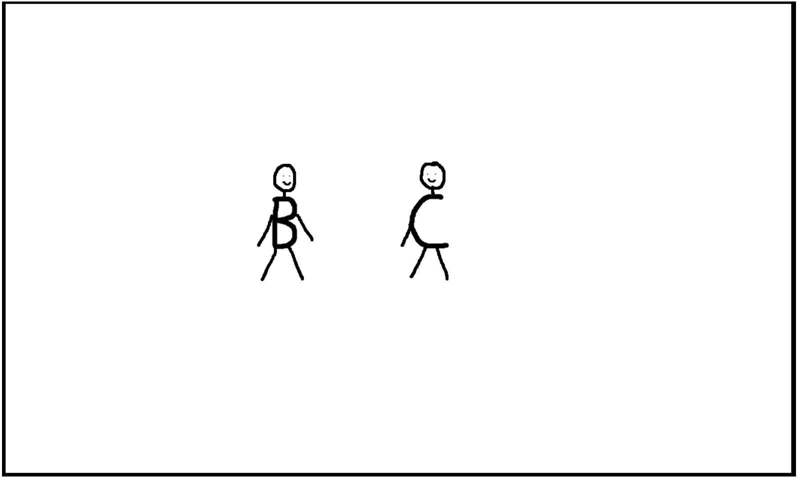

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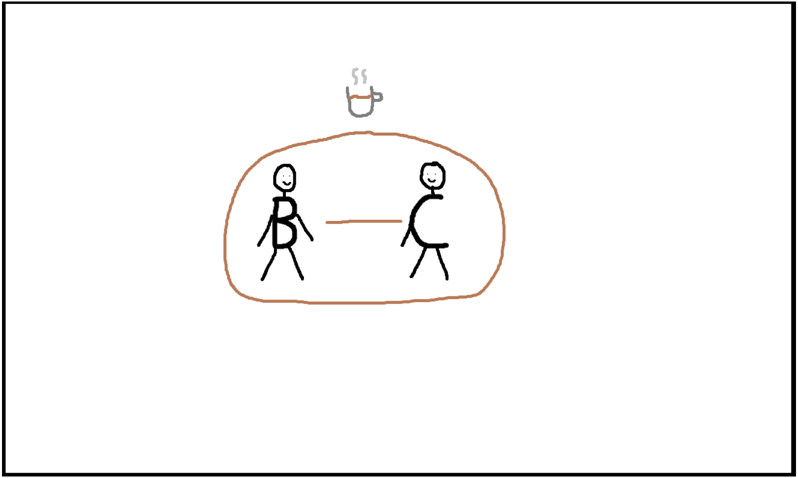

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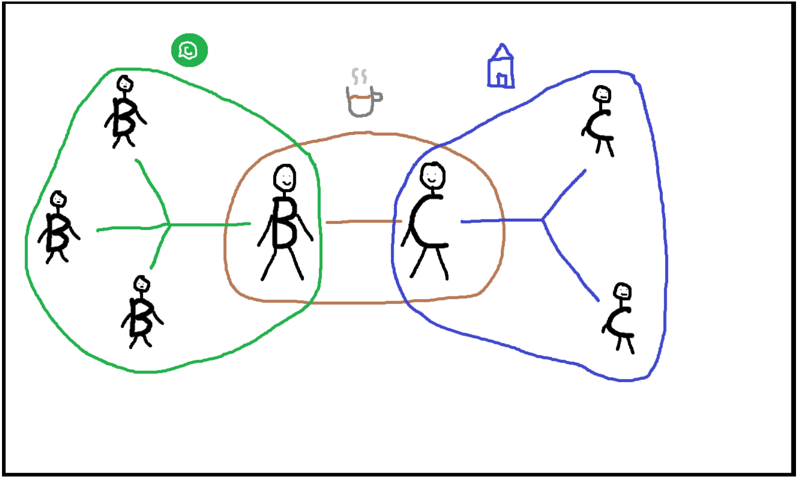

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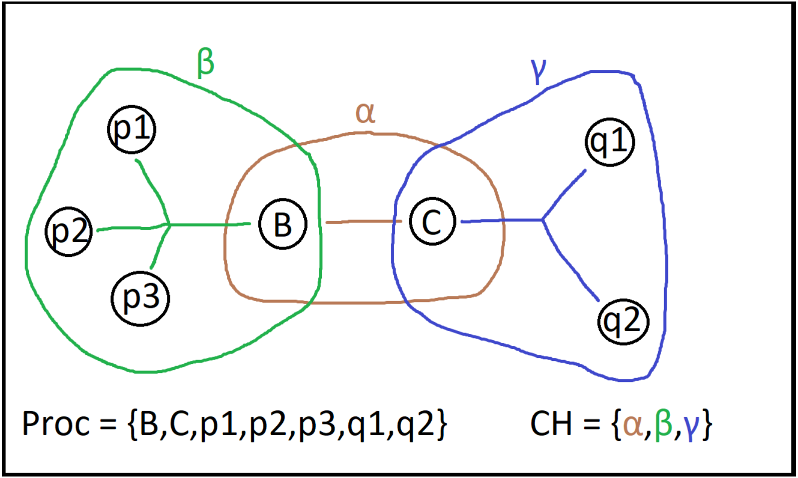

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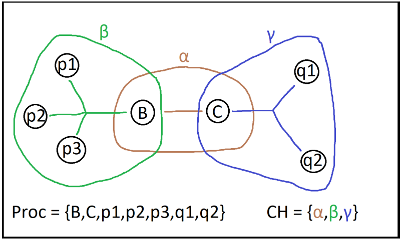

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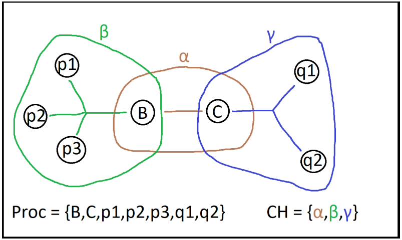

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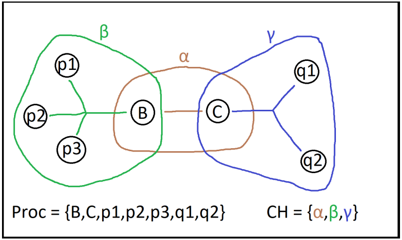
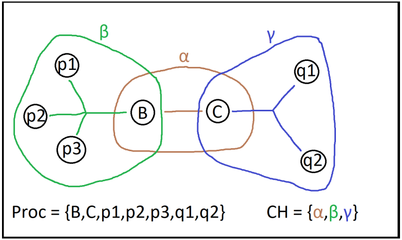
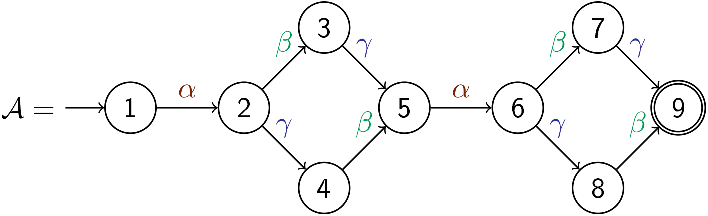


$$\mathcal{L} = \{\alpha \cdot \beta \cdot \gamma \cdot \alpha \cdot \beta \cdot \gamma, \\ \alpha \cdot \gamma \cdot \beta \cdot \alpha \cdot \beta \cdot \gamma, \\ \alpha \cdot \beta \cdot \gamma \cdot \alpha \cdot \gamma \cdot \beta, \\ \alpha \cdot \gamma \cdot \beta \cdot \alpha \cdot \gamma \cdot \beta\}$$

Example of a distributed protocol

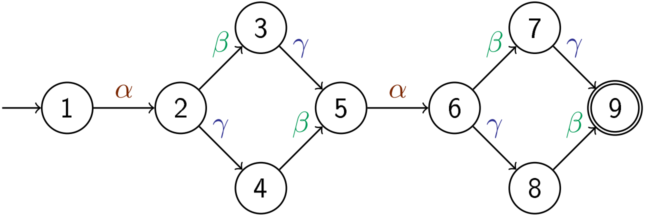


$$\mathcal{L} = \{\alpha\cdot\beta\cdot\gamma\cdot\alpha\cdot\beta\cdot\gamma,\ \alpha\cdot\gamma\cdot\beta\cdot\alpha\cdot\beta\cdot\gamma,\ \alpha\cdot\beta\cdot\gamma\cdot\alpha\cdot\gamma\cdot\beta,\ \alpha\cdot\gamma\cdot\beta\cdot\alpha\cdot\gamma\cdot\beta\}$$



Distributed view

$\mathcal{A} =$



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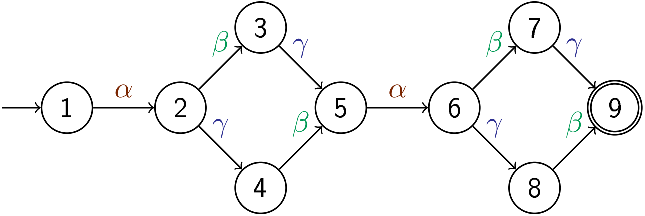
$\mathcal{A} =$ 

$\rho = \varepsilon$

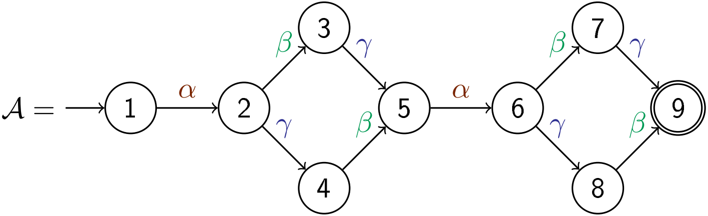
B : 1

C : 1

p_1 : 1

Actual state: 1

Distributed view



$\rho = \alpha$

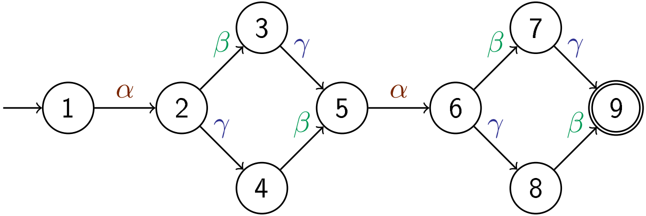
$$B : 1 \xrightarrow{\alpha} 2$$

$$C : 1 \xrightarrow{\alpha} 2$$

$$p_1 : 1$$

$$\text{Actual state: } 1 \xrightarrow{\alpha} 2$$

Distributed view

$\mathcal{A} =$ 

$\rho = \alpha \cdot \beta$

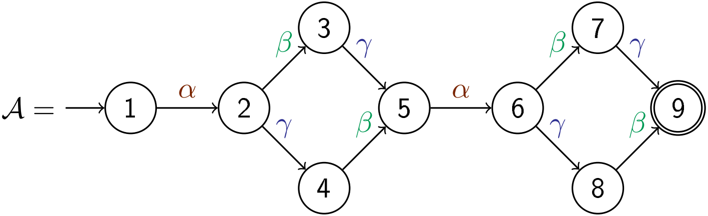
$$B : 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

$$C : 1 \xrightarrow{\alpha} 2$$

$$p_1 : 1 \xrightarrow{\beta} 3$$

$$\text{Actual state: } 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

Distributed view



$\rho = \alpha \cdot \beta \cdot \gamma$

$$B : 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$$

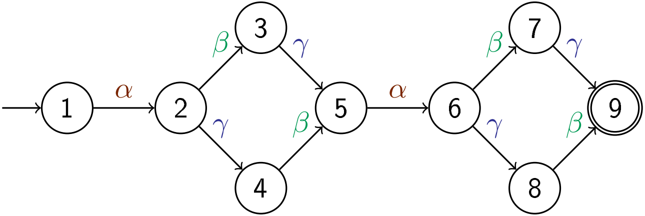
$$C : 1 \xrightarrow{\alpha} 2 \xrightarrow{\gamma} 4$$

$$p_1 : 1 \xrightarrow{\beta} 3$$

Actual state: $1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\gamma} 5$

Distributed view

$\mathcal{A} =$



$$\rho = \alpha \cdot \beta \cdot \gamma \cdot \alpha$$

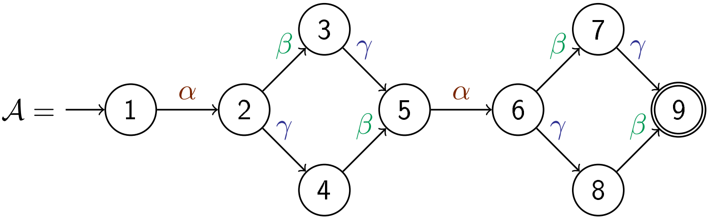
$$B : 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\alpha} 6$$

$$C : 1 \xrightarrow{\alpha} 2 \xrightarrow{\gamma} 4 \xrightarrow{\alpha} 6$$

$$p_1 : 1 \xrightarrow{\beta} 3$$

$$\text{Actual state: } 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\gamma} 5 \xrightarrow{\alpha} 6$$

Distributed view



$\rho = \alpha \cdot \beta \cdot \gamma \cdot \alpha \cdot \beta$

$$B : 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\alpha} 6 \xrightarrow{\beta} 7$$

$$C : 1 \xrightarrow{\alpha} 2 \xrightarrow{\gamma} 4 \xrightarrow{\alpha} 6$$

$$p_1 : 1 \xrightarrow{\beta} 3 \xrightarrow{\beta} 7$$

$$\text{Actual state: } 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\gamma} 5 \xrightarrow{\alpha} 6 \xrightarrow{\beta} 7$$

Distributed view

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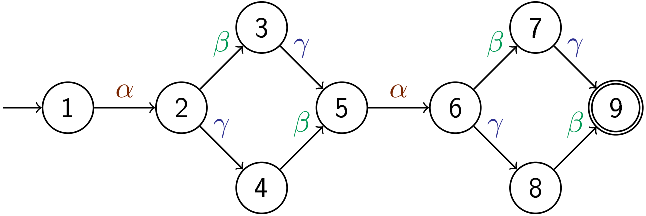

$\rho = \alpha \cdot \beta \cdot \gamma \cdot \alpha \cdot \beta \cdot \gamma$

$$B : 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\alpha} 6 \xrightarrow{\beta} 7$$

$$C : 1 \xrightarrow{\alpha} 2 \xrightarrow{\gamma} 4 \xrightarrow{\alpha} 6 \xrightarrow{\gamma} 8$$

$$p_1 : 1 \xrightarrow{\beta} 3 \xrightarrow{\beta} 7$$

$$\text{Actual state: } 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3 \xrightarrow{\gamma} 5 \xrightarrow{\alpha} 6 \xrightarrow{\beta} 7 \xrightarrow{\gamma} 9$$

Zielonka's distributivity theorem

Theorem [Zielonka, 1987]

Every diamond-closed language can be recognized by an asynchronous automaton.

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Theorem [Zielonka, 1987]

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- Complexity: exp. in $|Proc|$, poly. in $|\mathcal{A}|$ [Genest et al., 2010]
- On trees: $O(|\mathcal{A}|^2)$ construction [Krishna & Muscholl, 2013]

Reconfiguration

▶ What if processes could change dynamically the channels they listen to? Applications in:

- Swarm protocols (connected based on distance)
- Privacy (need-to-know)
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Reconfiguration

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- Adapt Zielonka's result to this setting:
  - Input language contains instructions for reconfiguration
  - Output automaton should implement them only with local information

Our result

Theorem

Every diamond-closed language with reconfiguration operations[2] and over a tree-like communication architecture[1] can be recognized by a reconfigurable[3] asynchronous automaton.

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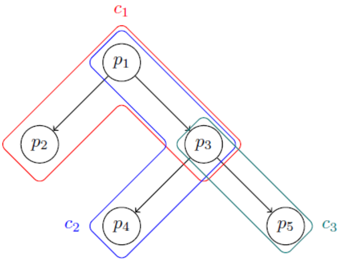

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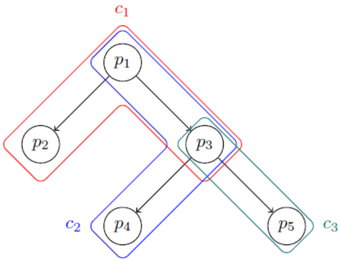

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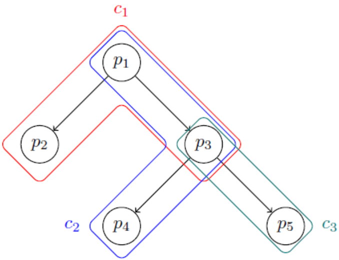

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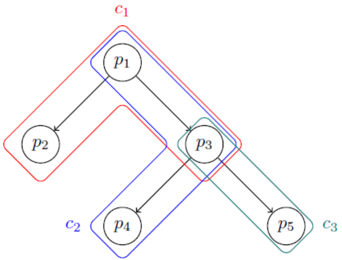

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- Complexity: $O(|\mathcal{A}|^2 \cdot 2^{|\mathrm{CH}|})$

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Adapted tree construction for Zielonka's distributivity theorem to the reconfigurable setting.

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Thanks, questions?